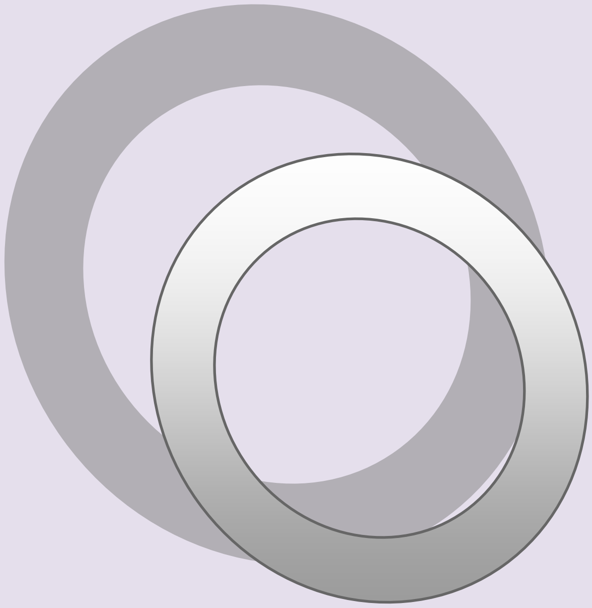
THE POWER SERIES IN POLYNOMIAL RING



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PREFACE

Firstly , we shall define the ring and take example on it , and define commutative ring,and then we define the sub ring and take example on it .

Finally , we shall define the power series in polynomial ring with take some principle theorems .

Definition 1 :-

Aset R is said to be ring if there exist two binary operations called Addition (+) and multiplication (.) They must satisfy the following :-

Axioms for addition

(0) For all a, b ϵ R, a + b ϵ R .
(1) a + (b + c) = (a + b) + c for all a, b and c belong to R .
(2) There exists 0 ϵ R such that a + 0 = 0 + a = a for all a ϵ R
(3) For all a ϵ R, there exists – a ϵ R with a+(-a) = (-a) +a = 0.

Axioms for multiplication

(0) For all a, b ϵ R then a.b ϵ R.
(1) a(b.c) = (a.b)c for all a, b and c belong to R.
(2) There exists 1 ϵ R such that a1 = 1a = a for all a ϵ R.

Mixed axiom

for all a, b and c belong to R then (a + b)c = a.c + b.c
and c(a + b) = c.a + c.b .

definition 2 :-

let R be aring then R is said to be commutative ring if for all a,b ϵ R$\Longrightarrow$ a b = b a .

Examples : -

(R ,+,.) , (Q,+,.) and (Z,+,.) are rings .

Definition 3 : -

let R be a ring , if $\emptyset \neq S \subseteq R$ is also ring then S is said to be sub ring of a ring R .

for example , Z is a sub ring of Q, and both are sub rings of R . such that $Z \subseteq Q$ & $Q \subseteq R$.

Definition 4 : -

A sequence $\sigma = (s0, s1, \ldots, si, \ldots)$ in a commutative ring R ($si \in R$) is called a polynomial if there is some integer $m \geq 0$ with $si = 0$ for all $i > m$

that is $\sigma = (s0, s1, \ldots, sm, 0, 0, \ldots)$.

Definition 5.

If $\sigma = (s0, s1, \ldots, sn, 0, 0, \ldots) \neq 0$ is a polynomial, then there is $sn \neq 0$

with $si = 0$ for all $i > n$. We call sn the **leading coefficient** of σ & we call n **the degree** of σ , and we denote the degree n by **deg (σ).**

Theorem 1 .

If $\sigma = (a_0, a_1, \ldots, a_n, 0, 0, \ldots)$, then $\sigma = a_0 + a_1 x + \cdots + a_n x^n$ where each element $a \in R$ is identified with the polynomial $(a, 0, 0, \ldots)$.

Proof.

$\sigma = (a_0, a_1, \ldots, a_n, 0, 0, \ldots)$

$= (a_0, 0, 0, \ldots) + (0, a_1, 0, \ldots) + \cdots + (0, 0, \ldots, a_n, 0, \ldots)$

$= a_0(1, 0, 0, \ldots) + a_1(0, 1, 0, \ldots) + \cdots + a_n(0, 0, \ldots, 1, 0, \ldots)$

$= a_0 + a_1 x + a_2 x^2 + \cdots + a_n x^n$

Definition 5:-

If R is a ring , then **the polynomial ring** R[x] is the ring of all polynomials in x with coefficients in R.

conceder a polynomial (equation 1) in x with coefficients in aring R as symbol

$$a_0 + a_1x^1 + a_2x^2 + \cdots + a_nx^n \ldots\ldots\ldots \quad (1)$$

If we add and multiply these symbols with usual way we get

a polynomial ring R[x] , such that R[x] is the set of all mappings from the set { $x^o, x^1, x^2, \ldots$ } into the ring R , add and multiply gives by the rules :

$$(f+g)(x^n) = f(x^n) + g(x^n) \quad \ldots\ldots(2)$$

$(fg)($

And the associative law of multiplication given by :

$((fg)$

$= \sum_{i+j=}$

$= \sum_{i+j=}$

$= \sum_{k+i=}$

Theorem 2.

If R is a commutative ring, then R[x] is a commutative ring that contains R as a sub ring.

Proof :

Define addition and multiplication of polynomials as follows:

If σ = (s0, s1, . . .) and τ = (t0, t1, . . .), then

σ + τ = (s0 + t0, s1 + t1, . . . , sn + tn, . . .)

and

στ = (c0, c1, c2, . . .),

where $ck = \sum_{i+j=k} s_i t_j = \sum_{i=0}^{k} s_i t_{k-i}$

identify with R.

Lemma 1.

Let R be a commutative ring and let σ , τ ∈ R[x] be nonzero polynomials either σ τ = 0 or deg(σ τ) ≤ deg(σ) + deg(τ).

Note :- let P = $\sum_{i=1}^{n} a_i x^i$ be a power series , let R be a ring if the coefficients of P belong to R then the members of R shall be as a power series , and the positive integer **n** is the degree of P .

Theorem 3 .

The power series $\sum_{i=0}^{n} a_i x^i$ & $\sum_{i=0}^{m} t_i x^i$ of degrees n and m, respectively, are equal if and only if n = m and $a_i = t_i$ for all i .

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